SHORE

Examination Number:

Set:

Year 12
Mathematics
Trial HSC Examination
August 2018

General instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"
- A NESA reference sheet is provided

Total marks – 100

Section I **Pages 2 – 5**

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II **Pages 6 – 13**

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which expression is a factorisation of $27x^3 - 8$?

A. $(3x-2)(9x^2-6x+4)$

B. $(3x-2)(9x^2+6x+4)$

C. $(3x-2)(9x^2-12x+4)$

D. $(3x-2)(9x^2+12x+4)$

2 The first three terms of an arithmetic series are 4, 7 and 10.
What is the sum of the first 18 terms?

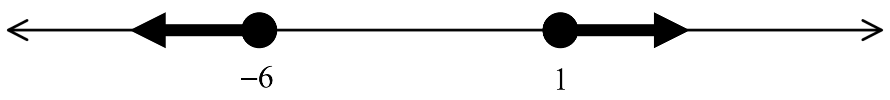
A. 55

B. 495

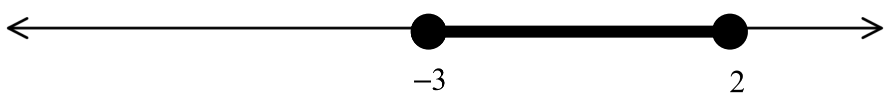
C. 531

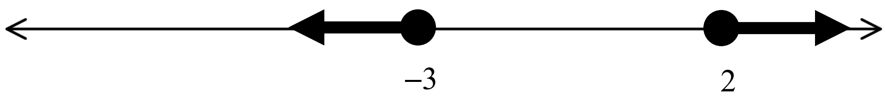
D. 558

3 What is the solution to $x^2 + 5x - 6 \geq 0$ when graphed on the number line?

A.


B.


C.


D.


4 The following two statements are made about the function $f(x) = \dfrac{3x^3}{x-2}$.

Statement 1: The function is discontinuous at $x = 2$.
Statement 2: The function is an odd function.

Which of the following is true?

A. Statement 1 and Statement 2 are both correct.

B. Statement 1 is correct and Statement 2 is incorrect.

C. Statement 1 is incorrect and Statement 2 is correct.

D. Statement 1 and statement 2 are both incorrect.

5 Using Simpson's rule with 5 function values, which expression gives you the area under the curve $y = x\ln x$ between $x = 2$ and $x = 4$?

A. $\frac{1}{4}\left(2\ln 2 + 5\ln 2.5 + 6\ln 3 + 7\ln 3.5 + 4\ln 4\right)$

B. $\frac{1}{6}\left(2\ln 2 + 5\ln 2.5 + 6\ln 3 + 7\ln 3.5 + 4\ln 4\right)$

C. $\frac{1}{4}\left(2\ln 2 + 10\ln 2.5 + 6\ln 3 + 14\ln 3.5 + 4\ln 4\right)$

D. $\frac{1}{6}\left(2\ln 2 + 10\ln 2.5 + 6\ln 3 + 14\ln 3.5 + 4\ln 4\right)$

6 For a particular angle, θ, $\cos\theta = -\dfrac{3}{\sqrt{34}}$ and $\tan\theta < 0$. What is the value of $\operatorname{cosec}\theta$?

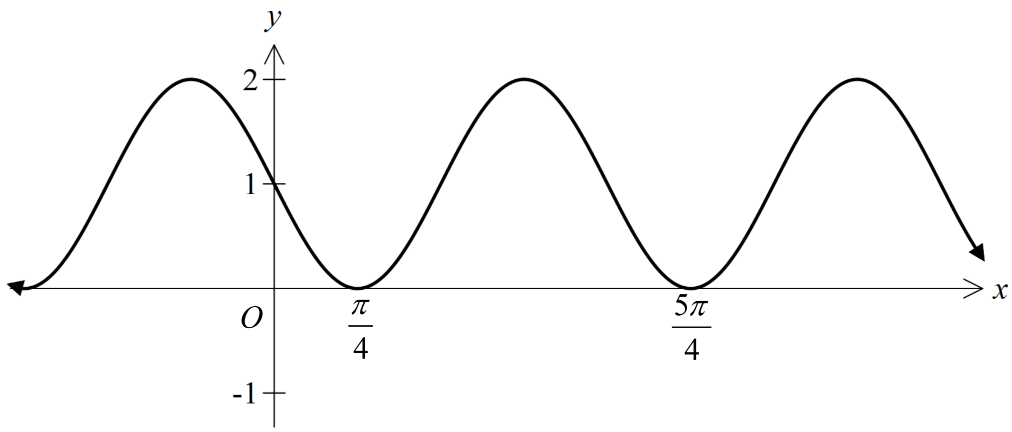
A. $\dfrac{5}{3}$

B. $-\dfrac{\sqrt{34}}{5}$

C. $\dfrac{\sqrt{34}}{5}$

D. $-\dfrac{3}{5}$

7 The curve below is of the form $y = 1 + a\sin(bx)$ where a and b are constants.



What are the values of a and b?

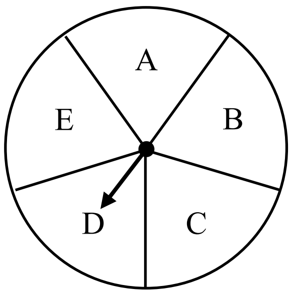
A. $a = 1$ and $b = 2$

B. $a = -1$ and $b = 2$

C. $a = -2$ and $b = 4$

D. $a = -1$ and $b = 4$

8 A spinner is marked with the letter A, B, C, D and E. When it is spun, each of the five letters is equally likely to occur.



The spinner is spun three times. What is the probability that a consonant appears on at least one of the three spins?

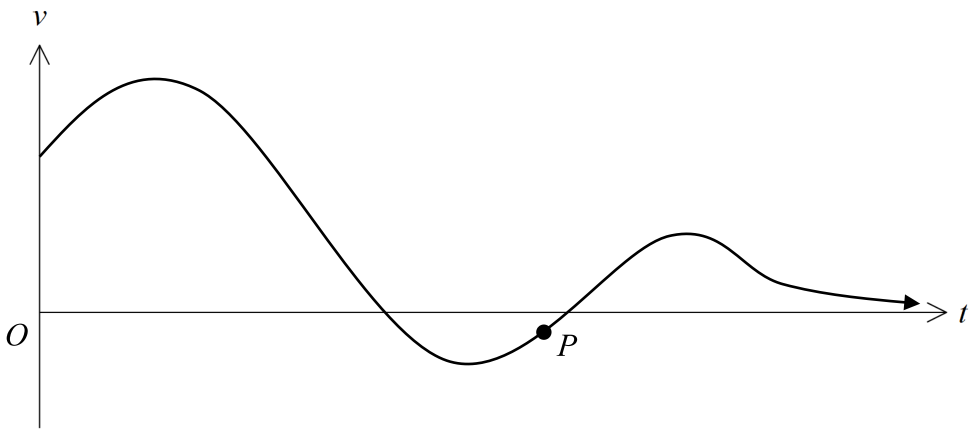
A. $\frac{8}{125}$

B. $\frac{27}{125}$

C. $\frac{3}{5}$

D. $\frac{117}{125}$

9 A particle initially starts at the origin. The graph, drawn to scale below, shows the velocity, v, of the particle moving along a straight line as a function of time, t.



Which statement describes the motion of the particle at point P?

A. It is to the left of the origin with positive acceleration.

B. It is to the left of the origin with negative acceleration.

C. It is to the right of the origin with positive acceleration.

D. It is to the right of the origin with negative acceleration.

10 The roots of the equation $px^2 + qx + r = 0$ are α and β.

What is the simplified expression for $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$?

A. $\dfrac{q^2 - 2pr}{r^2}$

B. $\dfrac{q^2 - 2p}{r}$

C. $\dfrac{q^2 - pr}{r^2}$

D. $\dfrac{r^2 - 2pq}{q^2}$

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11−16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Simplify $3x - 4x(2 + 5y)$. **1**

(b) Express $\dfrac{\sqrt{6}}{5-\sqrt{3}}$ with a rational denominator. **2**

(c) Differentiate $(\cos x + 2)^5$. **2**

(d) Differentiate $\dfrac{e^{3x}}{2x}$. **2**

(e) Find $\int (1 + \sec^2 3x)\, dx$. **2**

(f) A parabola has equation $(x-2)^2 = 20(y+3)$. Find the coordinates of the focus. **2**

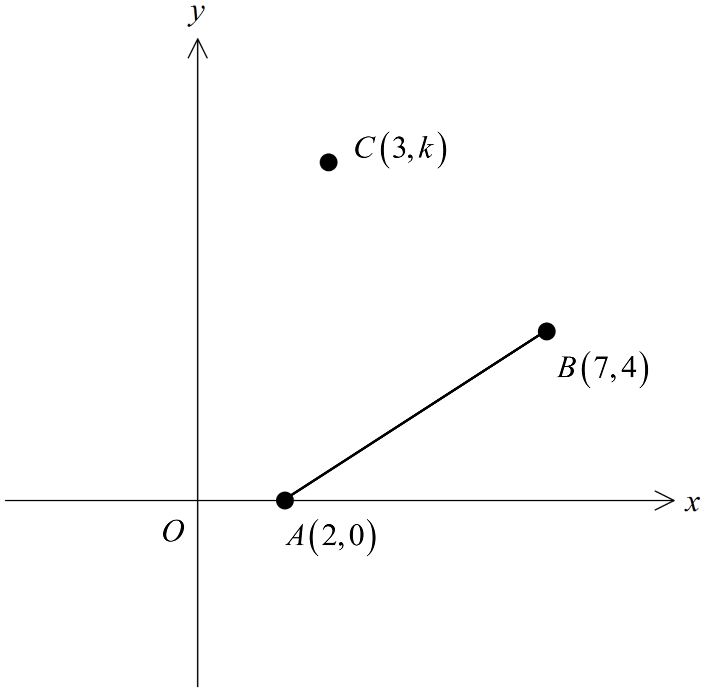
(g) Find the limiting sum of the infinite geometric series $\dfrac{7}{3} + \dfrac{14}{9} + \dfrac{28}{27} + \dots$. **2**

(h) Write down the domain and range of the function $y = \dfrac{1}{\sqrt{9-x}}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Find the gradient of the normal to the curve $y = \log_e\left(x^2 - 3\right)$ at the point $(2,0)$. **3**

(b) The points $A(2,0)$, $B(7,4)$ and $C(3,k)$ lie on a number plane.



(i) Show that the equation of line AB is $4x - 5y - 8 = 0$. **2**

The point C has coordinates $(3,k)$, where $k > 0$ and the perpendicular distance from C to AB is $\sqrt{41}$.

(ii) Show k can be found using the equation $41 = |4 - 5k|$ and hence find the value of k. **3**

(iii) Prove triangle ABC is right angled. **2**

(c) Evaluate $\displaystyle\int_1^8 \frac{1}{\sqrt{x}}\,dx$, leaving your answer in the form $a + b\sqrt{2}$, where a and b are integers. **3**

(d) Sketch the region defined by $y \geq \dfrac{2}{x+1}$. **2**

Question 13 (15 marks) Use a SEPARATE writing booklet

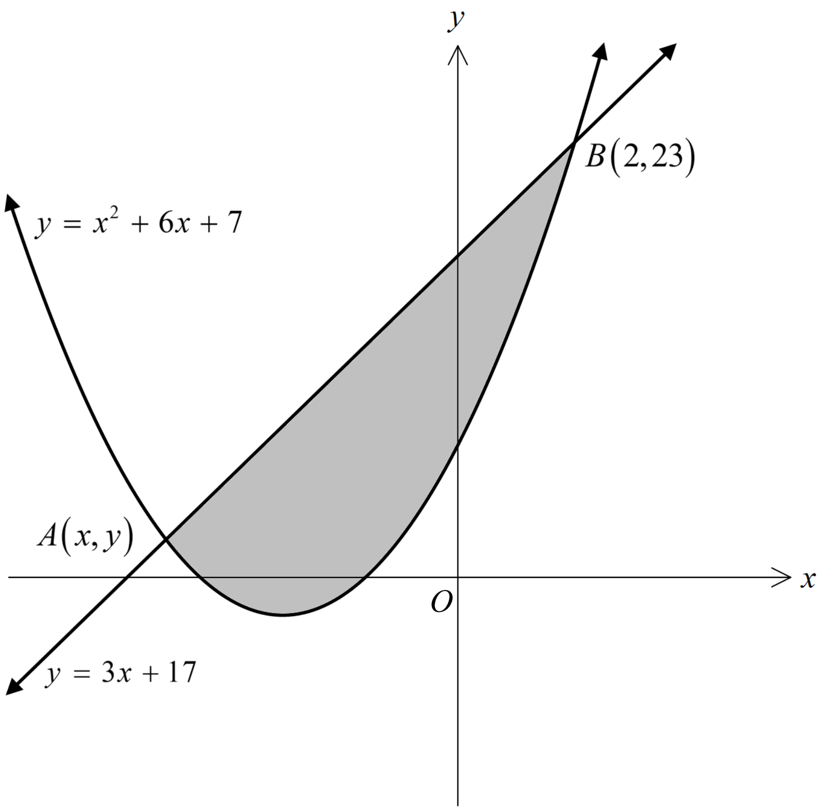
(a) Find $\displaystyle\int \frac{x}{x^2+3}\,dx$. **2**

(b) Points A and B have coordinates $(4,-1)$ and $(12,11)$ respectively.

(i) Find the coordinates of midpoint AB. **1**

(ii) Given AB is the diameter of a circle, find the equation of the circle. **2**

(c) The line $y = 3x + 17$ intersects the curve $y = x^2 + 6x + 7$ at the points $A(x, y)$ and $B(2, 23)$.



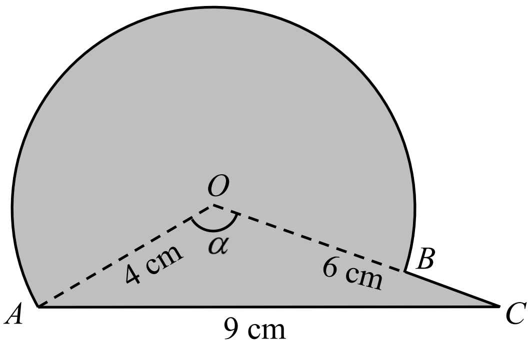
(i) Find the x coordinate of point A. **2**

(ii) Find the exact area bounded by the line and the curve. **3**

Question 13 continues on page 9

Question 13 (continued)

(d) In the figure below $OC = 6$ cm, $AC = 9$ cm, $AO = 4$ cm and $\angle AOB = \alpha$.



(i) Show that, correct to 2 decimal places, $\alpha = 2.22$ radians. **2**

(ii) Find the area of the shade shape, correct to 1 decimal place. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

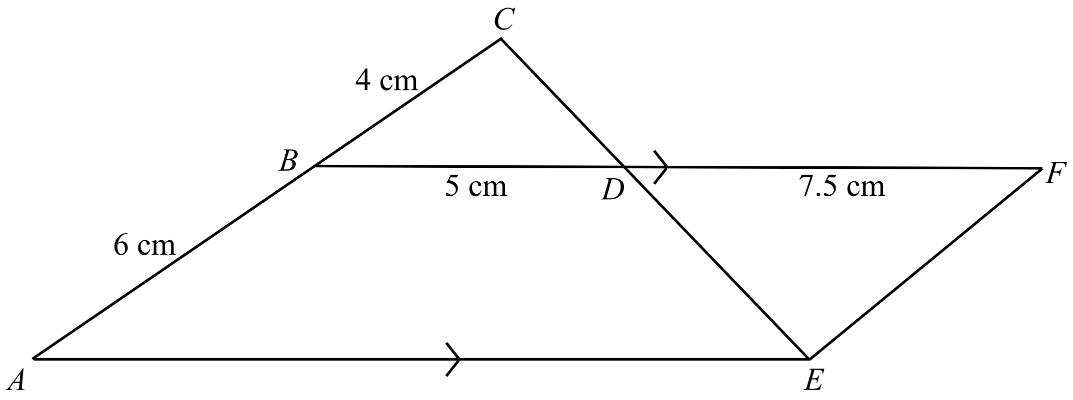
(a) Consider the curve $y = -x^3 + 3x^2 - 4$.

(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Find the coordinates of any point of inflexion. **2**

(iii) Sketch the curve, labelling the y-intercept, stationary points and any point of inflexion. Do not find the x-intercepts. **2**

(b) In the diagram below $BF \parallel AE, AB = 6\,\text{cm}, BC = 4\,\text{cm}, BD = 5\,\text{cm}$ and $DF = 7.5\,\text{cm}$.



(i) Show that $DE = \dfrac{3CD}{2}$, giving reasons. **2**

(ii) Given $CD = 2\,\text{cm}$, prove $\Delta BCD \;|||\; \Delta FED$. **2**

(c) A company predicts a yearly profit of \$140 000 in the year 2018. The company predicts that the yearly profit will rise each year by 5%.

(i) Show the predicted profit in the year 2020 is given by $140\,000 \times 1.05^2$. **1**

(ii) Find the total predicted profit for the years 2018 to 2030 inclusive, giving your answer to the nearest dollar. **3**

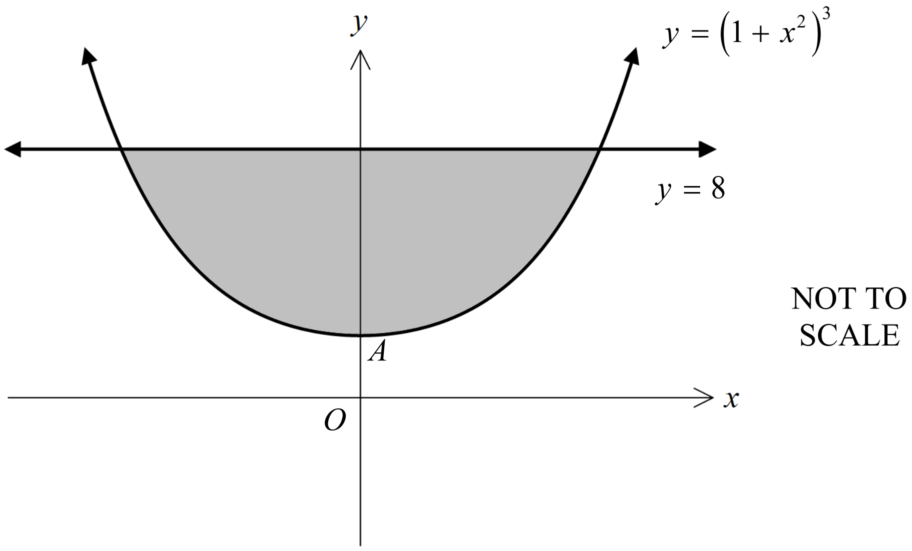
Question 15 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\displaystyle\int_1^e \frac{x^2+1}{x}\,dx$. **3**

(b) Solve $2\cos^2 x - 7\sin x + 2 = 0$ for $0 \le x \le 2\pi$. **3**

(c) The region enclosed by $y = \left(1+x^2\right)^3$ and the line $y = 8$ is shown below.

The curve $y = \left(1+x^2\right)^3$ intersects the y-axis at point A.



(i) Find the coordinates of point A. **1**

(ii) The region is rotated about the y-axis. Find the volume of the solid formed, leaving your answer in terms of π . **3**

(d) Consider the equation $2\log_2(x+15) - \log_2 x = 6$.

(i) Show that $x^2 - 34x + 225 = 0$. **3**

(ii) Hence, or otherwise, solve the equation $2\log_2(x+15) - \log_2 x = 6$. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) The velocity, v ms^{-1}, of a particle moving along a straight line is given by $v = 3t^2 - 12t + 14$, where t is the time in seconds.

(i) Find the initial velocity of the particle. **1**

(ii) Show that v is positive for all values of t. **2**

(iii) Find the distance travelled between the times $t = 1$ and $t = 3$. **2**

(b) A scientist is researching the growth of a certain species of hamster. She proposes that the length, x cm, of a hamster t days after birth is given by $x = 15 - 12e^{-\frac{t}{14}}$.

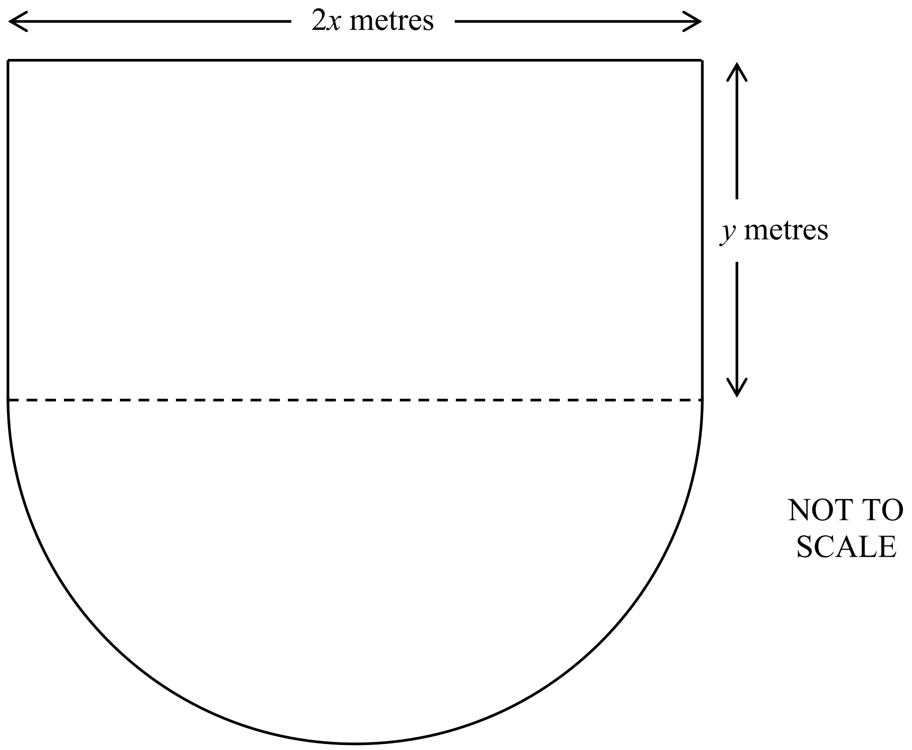
(i) Find the length of a hamster when it is born. **1**

(ii) Find the number of days it takes for a hamster to grow to 10 cm in length, correct to 1 decimal place. **2**

(iii) Find the rate of growth of the hamster 8 days after birth. Give your answer in cm per day, correct to 1 decimal place. **2**

Question 16 continues on page 13

(c) A stage, with a perimeter of 80 metres is made by joining a rectangle and a semicircle. The length of the rectangle is $2x$ metres and the width of the rectangle is y metres. The diameter of the semicircle is $2x$ metres.



(i) Show that the area, A m^2, of the stage is given by $A = 80x - \left(2 + \frac{\pi}{2}\right)x^2$. **2**

(ii) Find the value of x, correct to 3 significant figures, that will provide the maximum stage area. Justify why this value of x gives the maximum area. **3**

End of paper

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2018 - TRIAL HSC - MATHEMATICS

1 B 2 C 3 A 4 B 5 D 6 C 7 B 8 D 9 C 10 A

1 $27x^3 - 8 = (3x)^3 - 2^3$

$= (3x-2)(9x^2 + 6x + 4)$

$\therefore B$

2 $4, 7, 10 \quad \therefore a = 4,\ d = 3$

$S_{18} = \frac{18}{2}\left(2(4) + (18-1)\times 3\right)$

$= 9 \times 59$

$= 531$

$\therefore C$

3 $x^2 + 5x - 6 \geq 0$

crit. pts $\quad x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$\therefore x = -6 \quad$ or $\quad x = 1$

$\therefore x \leq -6 \quad$ or $\quad x \geq 1$

$\therefore A$

4 $f(x) = \frac{3x^3}{x-2}$

$x \neq 2 \ \therefore$ statement 1 true

$f(-x) = \frac{3(-x)^3}{-x-2}$

$= \frac{-3x^3}{-x-2}$

$\neq -f(x)$

$\therefore$ statement 2 is false

$\therefore B$

5 $y = x\ln x$

x	2	2.5	3	3.5	4
y	$2\ln 2$	$2.5\ln 2.5$	$3\ln 3$	$3.5\ln 3.5$	$4\ln 4$
	1	4	2	4	1

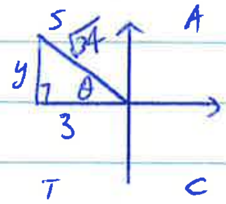
$h = \frac{1}{2}$

$A = \frac{\frac{1}{2}}{3}\left[2\ln 2 + 4(2.5\ln 2.5 + 3.5\ln 3.5) + 2 \cdot 3\ln 3 + 4\ln 4\right]$

$= \frac{1}{6}\left[2\ln 2 + 10\ln 2.5 + 6\ln 3 + 14\ln 3.5 + 4\ln 4\right]$

$\therefore D$

6 $\cos\theta = -\frac{3}{\sqrt{34}},\ \tan\theta < 0$



$y = \sqrt{(\sqrt{34})^2 - 3^2}$

$= \sqrt{25}$

$= 5$

$\therefore \sin\theta = \frac{5}{\sqrt{34}}$

$\therefore \operatorname{cosec}\theta = \frac{\sqrt{34}}{5}$

$\therefore C$

7 $y = 1 + a\sin(bx)$

- flipped with amplitude 1 $\therefore a = -1$
- repeats after π $\therefore b = 2$

$\therefore B$

8 $P_{(\text{consonant})} = 1 - P_{(\text{no consonants})}$

$= 1 - \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right)$

$= \frac{117}{125}$

$\therefore D$

9 negative velocity but still right of origin
positive acceleration

$\therefore C$

(10) $\alpha+\beta = -\frac{q}{p}$ $\qquad \alpha\beta = \frac{r}{p}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\frac{q^2}{p^2} - \frac{2r}{p}}{\frac{r^2}{p^2}}$$

$$= \frac{q^2-2pr}{p^2} \div \frac{r^2}{p^2}$$

$$= \frac{q^2-2pr}{r^2}$$

Question 11

11 a) $3x - 4x(2+5y) = 3x - 8x - 20xy$
$= -5x - 20xy$

b) $\dfrac{\sqrt{6}}{5-\sqrt{3}} = \dfrac{\sqrt{6}}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}}$

$= \dfrac{5\sqrt{6}+\sqrt{18}}{5^2-(\sqrt{3})^2}$

$= \dfrac{5\sqrt{6}+3\sqrt{2}}{22}$

c) $\dfrac{d}{dx}(\cos x+2)^5 = 5(\cos x+2)^4 \times -\sin x$
$= -5\sin x(\cos x+2)^4$

d) $\dfrac{d}{dx}\dfrac{e^{3x}}{2x} = \dfrac{vu'-uv'}{v^2}$ $\quad u = e^{3x}$, $u' = 3e^{3x}$ $\quad v = 2x$, $v' = 2$

$= \dfrac{2x \times 3e^{3x} - 2 \times e^{3x}}{(2x)^2}$

$= \dfrac{6xe^{3x} - 2e^{3x}}{4x^2}$

$= \dfrac{2e^{3x}(3x-1)}{4x^2}$

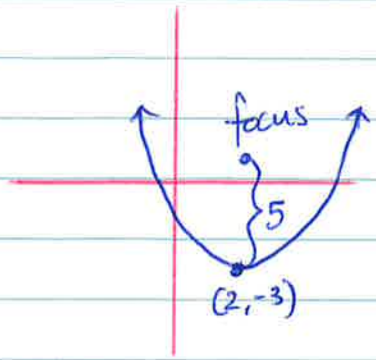
$= \dfrac{e^{3x}(3x-1)}{2x^2}$

e) $\displaystyle\int 1 + \sec^2 3x \, dx = x + \frac{1}{3}\tan 3x + c$

f) $(x-2)^2 = 20(y+3)$

$\therefore$ vertex $(2,-3)$ focal length $= 5$.

$\therefore$ focus $(2,2)$



g) $\frac{7}{3} + \frac{14}{9} + \frac{28}{27} + \ldots$

$a = \frac{7}{3}$ $\quad r = \frac{2}{3}$

$$\therefore S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{7}{3}}{1-\frac{2}{3}}$$

$$= 7$$

h) $y = \frac{1}{\sqrt{9-x}}$

domain $x : x < 9$

range $y : y > 0$

Question 12

12 a) $y = \log_e(x^2-3)$

$$y' = \frac{1}{x^2-3} \times 2x$$

$$= \frac{2x}{x^2-3}$$

at $x=2$, $y' = \dfrac{2(2)}{2^2-3}$

$$= 4$$

$\therefore M_{tangent} = 4$

$\therefore M_{normal} = -\frac{1}{4}$

b) i) $M_{AB} = \dfrac{4-0}{7-2}$ $\quad A(2,0)\quad B(7,4)\quad C(3,k)$

$$= \frac{4}{5}$$

$\therefore y-0 = \frac{4}{5}(x-2)$

$5y = 4(x-2)$

$5y = 4x-8$

$0 = 4x-5y-8$

ii) $d = \dfrac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$$\sqrt{41} = \frac{|4(3)+-5(k)+(-8)|}{\sqrt{4^2+(-5)^2}}$$

$$\sqrt{41} = \frac{|4-5k|}{\sqrt{41}}$$

$$41 = |4-5k|$$

$\therefore\ 41 = |4-5k|$

$4-5k = 41$ or $4-5k = -41$

$-5k = 37$ $\quad$ $-5k = -45$

$k = -\frac{37}{5}$ $\quad$ $k = 9$

$k > 0, \therefore k = 9$

iii) $m_{AB} = \frac{4}{5}$ $\quad$ $B(7,4)$ $\quad$ $C(3,9)$

$m_{BC} = \frac{9-4}{3-7}$

$= \frac{5}{-4}$

$= -\frac{5}{4}$

$m_{AB} \times m_{BC} = \frac{4}{5} \times -\frac{5}{4}$

$= -1$

$\therefore AB \perp BC$

$\therefore \Delta ABC$ is right angled

c) $\int_1^8 \frac{1}{\sqrt{x}}\,dx = \int_1^8 x^{-\frac{1}{2}}\,dx$

$= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^8$

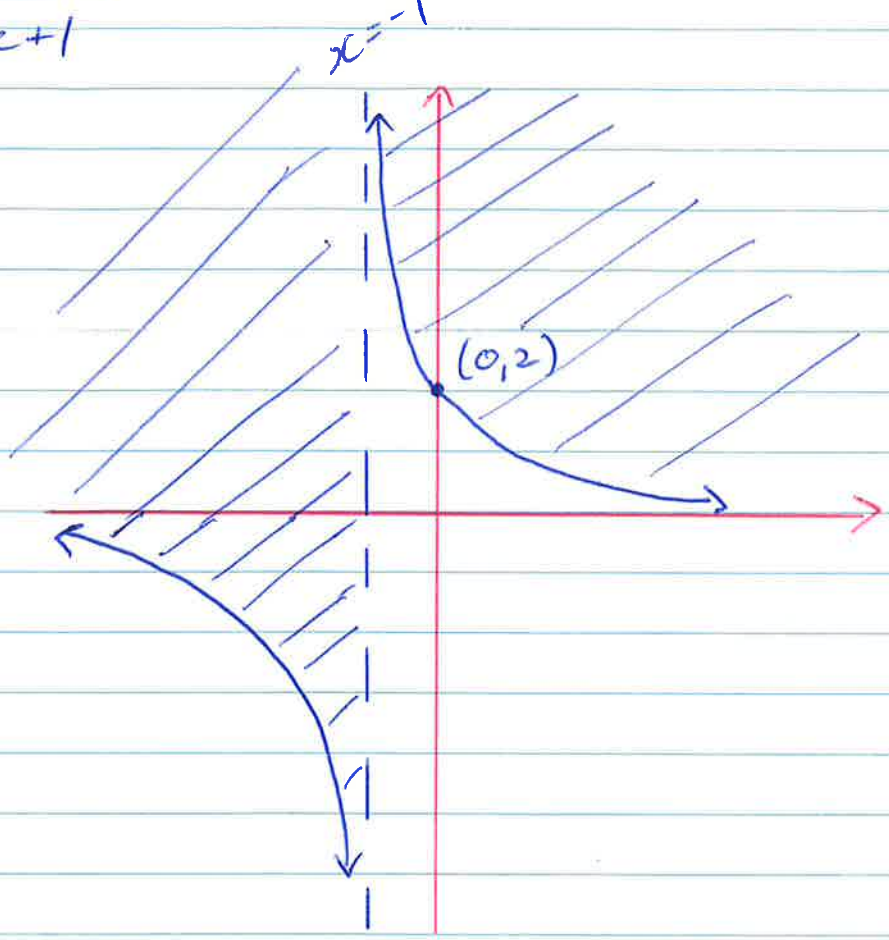
$= \left[2\sqrt{x}\right]_1^8$

$= (2\sqrt{8}) - (2\sqrt{1})$

$= 2\times2\sqrt{2} - 2$

$= -2 + 4\sqrt{2}$

d) $y \geq \dfrac{2}{x+1}$



Question 13

13 a) $\int \frac{x}{x^2+3}\,dx = \frac{1}{2} \times \int \frac{2x}{x^2+3}\,dx$

$= \frac{1}{2}\log_e(x^2+3) + C$

b) $A(4,-1)$ $B(12,11)$

i) $MP_{AB} = \left(\frac{4+12}{2}, \frac{-1+11}{2}\right)$

$= (8, 5)$

ii) distance B to MP $= \sqrt{(11-5)^2+(12-8)^2}$

$= \sqrt{52}$

$\therefore (x-8)^2 + (y-5)^2 = (\sqrt{52})^2$

$(x-8)^2 + (y-5)^2 = 52$

c) $y = 3x + 17$ & $y = x^2 + 6x + 7$

i) $x^2 + 6x + 7 = 3x + 17$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\therefore x = -5$ & $x = 2$

$\therefore$ x-coordinate of A is -5

ii) $\text{Area} = \int_{-5}^{2} \left[3x+17-(x^2+6x+7)\right] \, dx$

$$= \int_{-5}^{2} -x^2 - 3x + 10 \quad dx$$

$$= \left[-\frac{x^3}{3} - \frac{3x^2}{2} + 10x\right]_{-5}^{2}$$

$$= \left(-\frac{2^3}{3} - \frac{3(2)^2}{2} + 10(2)\right) - \left(-\frac{(-5)^3}{3} - \frac{3(-5)^2}{2} + 10(-5)\right)$$

$$= \frac{34}{3} - \left(-\frac{275}{6}\right)$$

$$= \frac{343}{6} u^2 \quad \text{or} \quad 57\tfrac{1}{6} u^2$$

d) i) $\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times4\times6}$

$\cos\alpha = -\dfrac{29}{48}$

$\alpha = \cos^{-1}\left(-\dfrac{29}{48}\right)$

$\alpha = 2.2195..$

$\alpha = 2.22$

ii) $\text{Area}_\Delta = \frac{1}{2}\times4\times6\times\sin 2.22$

$= 9.5587$

$\text{Area}_{\text{Sector}} = \frac{1}{2}\times4^2\times(2\pi-2.22)$

$= 32.505..$

$\therefore \text{Area} = 9.5587.. + 32.505..$

$= 42.1 \text{ cm}^2$

Question 14

14 a) $y = -x^3 + 3x^2 - 4$

i) stat pts when $y' = 0$

$y' = -3x^2 + 6x$

$0 = -3x^2 + 6x$

$0 = -3x(x-2)$

$\therefore x = 0$ or $x = 2$

$y'' = -6x + 6$

at $x = 0$ $y'' = -6(0) + 6$
$= 6$

$\therefore$ min t.p.

at $x = 2$ $y'' = -6(2) + 6$
$= -6$

$\therefore$ max t.p.

at $x = 0$ $y = -(0)^3 + 3(0)^2 - 4$
$= -4$

at $x = 2$ $y = -2^3 + 3(2)^2 - 4$
$= 0$

$\therefore (0, -4)$ is min t.p.

$\therefore (2, 0)$ is max. t.p.

ii) pt of inflexion at $y'' = 0$

$y'' = -6x + 6$

$0 = -6x + 6$

$6x = 6$

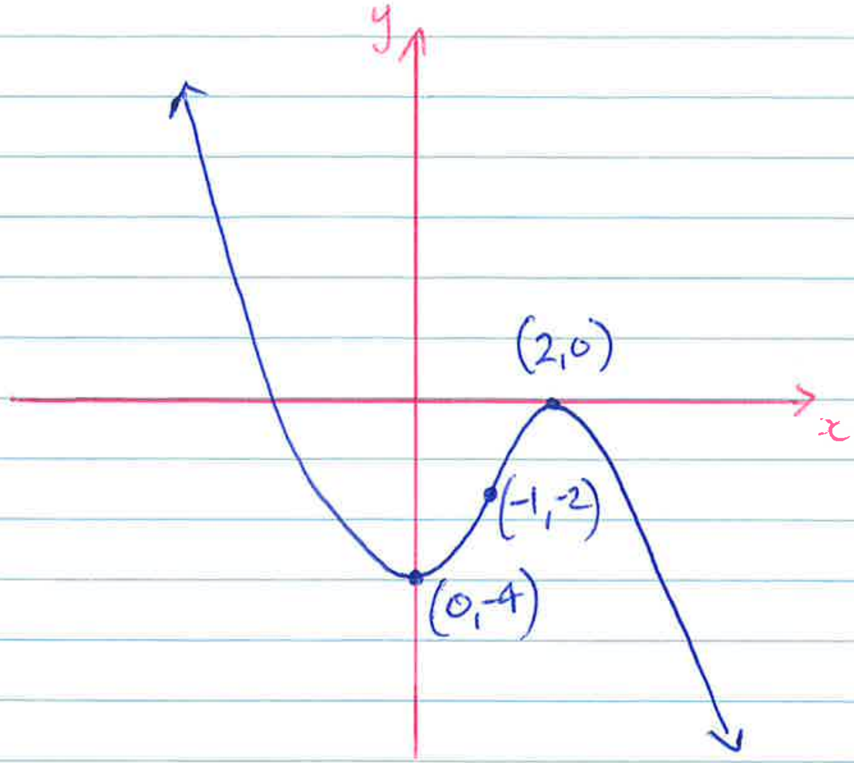
$x = 1$

at $x = 1$ $y = -1^3 + 3(1)^2 - 4$
$= -2$

x	0	1	2
y''	6	0	-6

$\therefore$ change in concavity

$\therefore (1, -2)$ is point of inflexion iii)



b) i) $\frac{DE}{CD} = \frac{AB}{BC}$ (ratio of intercepts)

$\frac{DE}{CD} = \frac{6}{4}$

$\frac{DE}{CD} = \frac{3}{2}$

$DE = \frac{3CD}{2}$

ii) if $CD = 2$ then $DE = \frac{3(2)}{2}$

$= 3\text{cm}$

In ΔBCD & ΔFED

- $\frac{DE}{CD} = \frac{3}{2}$
- $\angle CDB = \angle EDF$ (vertically opposite angles)
- $\frac{DF}{DB} = \frac{7.5}{5} = \frac{3}{2}$

$\therefore \Delta BCD \,|||\, \Delta FED$ (two matching sides in same ratio & included angle equ

c) i) $P_1 = 140000$ 2018

$P_2 = 140000 \times 1.05$ 2019

$P_3 = P_2 \times 1.05$ 2020

$= 140000 \times 1.05 \times 1.05$

$= 140000 \times 1.05^2$

ii) Note 2030 is P_{13}

Total profit $= P_1 + P_2 + P_3 + \dots P_{13}$

$= 140000 + 140000 \times 1.05 + 140000 \times 1.05^2 + \dots + 140000 \times 1.05^{12}$

$= 140000(1 + 1.05 + 1.05^2 + \dots + 1.05^{12})$

$a = 1, \quad r = 1.05, \quad n = 13$

$$S_{13} = \frac{1(1 - 1.05^{13})}{1 - 1.05}$$

$= 17.7129\dots$

$\therefore$ Total Profit $= 140000 \times 17.7129\dots$

$= 2479817.59\dots$

$= \$2479818$

Question 15

15 a) $\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e \frac{x^2}{x} + \frac{1}{x}\,dx$

$= \int_1^e x + \frac{1}{x}\,dx$

$= \left[\frac{x^2}{2} + \ln x\right]_1^e$

$= \left(\frac{e^2}{2} + \ln e\right) - \left(\frac{1^2}{2} + \ln 1\right)$

$= \frac{e^2}{2} + 1 - \frac{1}{2}$

$= \frac{e^2+2-1}{2}$

$= \frac{e^2+1}{2}$

b) $2\cos^2 x - 7\sin x + 2 = 0$

$\cos^2 x = 1 - \sin^2 x$

$\therefore 2(1-\sin^2 x) - 7\sin x + 2 = 0$

$2 - 2\sin^2 x - 7\sin x + 2 = 0$

$-2\sin^2 x - 7\sin x + 4 = 0$

$2\sin^2 x + 7\sin x - 4 = 0$

let $u = \sin x$

$2u^2 + 7u - 4 = 0$

P -8
S 7
F $8, -1$

$2u^2 + 8u - u - 4 = 0$

$2u(u+4) - 1(u+4) = 0$

$(u+4)(2u-1) = 0$

$\therefore u + 4 = 0$

$u = -4$

$\sin x = -4$

no solutions

$2u - 1 = 0$

$u = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

c) $y = (1 + x^2)^3$

i) let $x = 0$

$y = (1+0)^3$
$= 1$

$\therefore A(0,1)$

ii) $V = \pi \times \int_a^b [f(y)]^2 \, dy$

$y = (1+x^2)^3$

$\sqrt[3]{y} = 1 + x^2$

$x^2 = y^{\frac{1}{3}} - 1$

$\therefore V = \pi \times \int_1^8 y^{\frac{1}{3}} - 1 \, dy$

$= \pi \times \left[\frac{y^{\frac{4}{3}}}{\frac{4}{3}} - y \right]_1^8$

$= \pi \times \left[\frac{3y^{\frac{4}{3}}}{4} - y \right]_1^8$

$= \pi \times \left[\left(\frac{3(8)^{\frac{4}{3}}}{4} - 8 \right) - \left(\frac{3(1)^{\frac{4}{3}}}{4} - 1 \right) \right]$

$= \pi \times \left(4 - \left(-\frac{1}{4} \right) \right)$

$= \frac{17\pi}{4} \, u^3$

d) $2\log_2(x+15) - \log_2 x = 6$

i) $2\log_2(x+15) - \log_2 x = 6$

$$\log_2(x+15)^2 - \log_2 x = 6$$

$$\log_2 \frac{(x+15)^2}{x} = 6$$

$\log_B A = P$

$$\therefore 2^6 = \frac{(x+15)^2}{x}$$

$$64x = (x+15)^2$$

$$64x = x^2 + 30x + 225$$

$$0 = x^2 - 34x + 225$$

ii) $x^2 - 34x + 225 = 0$

$(x-9)(x-25) = 0$

P 225
S −34
F −9, −25

$\therefore x = 9$ or $x = 25$

Question 16

16 a) i) $V = 3t^2 - 12t + 14$

$V = 3(0)^2 - 12(0) + 14$

$= 14$ m/s

ii) $\Delta = b^2 - 4ac$

$= (-12)^2 - 4 \times 3 \times 14$

$= 144 - 168$

$= -24$

for $v = 3t^2 - 12t + 14$

$a > 0$ & $\Delta < 0$

$\therefore$ positive definite

iii) $x = \int v$

$= \int 3t^2 - 12t + 14 \, dt$

$= \frac{3t^3}{3} - \frac{12t^2}{2} + 14t +$

$= t^3 - 6t^2 + 14t + c$

at $t = 1$ $\quad x = 1^3 - 6(1)^2 + 14(1) + c$

$= 9 + c$

at $t = 3$ $\quad x = 3^3 - 6(3)^2 + 14(3) + c$

$= 15 + c$

$\therefore$ distance $= (15 + c) - (9 + c)$

$= 6$ m

b) $x = 15 - 12e^{-\frac{t}{14}}$

i) at $t=0$ $\quad x = 15 - 12e^{0}$

$= 15 - 12$

$= 3\text{cm}$

ii) $10 = 15 - 12e^{-\frac{t}{14}}$

$-5 = -12e^{-\frac{t}{14}}$

$\frac{5}{12} = e^{-\frac{t}{14}}$ $\qquad \log_B A = P$

$\log_e\left(\frac{5}{12}\right) = -\frac{t}{14}$

$t = -14 \log_e\left(\frac{5}{12}\right)$

$t = 12.256..$

$= 12.3$ days

iii) $x = 15 - 12e^{-\frac{t}{14}}$

$\frac{dx}{dt} = -12e^{-\frac{t}{14}} \times -\frac{1}{14}$

$= \frac{6e^{-\frac{t}{14}}}{7}$

at $t=8$ $\qquad \frac{dx}{dt} = \frac{6e^{-\frac{8}{14}}}{7}$

$= 0.484..$

$= 0.5$ cm/day

c) Area $= 2xy + \pi \times x^2 \times \frac{1}{2}$

$$= 2xy + \frac{\pi x^2}{2}$$

i) Perimeter $= \frac{1}{2} \times \pi \times 2x + 2x + 2y$

$$80 = \pi x + 2x + 2y$$

$$2y = 80 - \pi x - 2x$$

$$y = 40 - \frac{\pi x}{2} - x$$

$\therefore$ Area $= 2x\left(40 - \frac{\pi x}{2} - x\right) + \frac{\pi x^2}{2}$

$$= 80x - \pi x^2 - 2x^2 + \frac{\pi x^2}{2}$$

$$= 80x - 2x^2 + \frac{\pi x^2}{2} - \frac{2\pi x^2}{2}$$

$$= 80x - 2x^2 - \frac{\pi x^2}{2}$$

$$= 80x - x^2\left(2 + \frac{\pi}{2}\right)$$

$$= 80x - \left(2 + \frac{\pi}{2}\right)x^2$$

ii) $A = 80x - \left(2 + \frac{\pi}{2}\right)x^2$

$$\frac{dA}{dx} = 80 - 2\left(2 + \frac{\pi}{2}\right)x$$

Max/min when $\frac{dA}{dx} = 0$

$$0 = 80 - 2\left(2 + \frac{\pi}{2}\right)x$$

$$2\left(2 + \frac{\pi}{2}\right)x = 80$$

$$\left(2 + \frac{\pi}{2}\right)x = 40$$

$$x = \frac{40}{\left(2 + \frac{\pi}{2}\right)}$$

$$x = 11.2019\ldots$$

x	11	11.2	11.5
$\frac{dA}{dx}$	1.44	0	-2.12
	/	—	\

$\therefore$ Max area when $x = 11.2$ m